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Two-Phase Equilibria of Lipid-Bilayer Vesicles: Existence of Symmetry-Breaking Solutions and Their Computation

Tim Healey

Professor, Department of Mathematics, Cornell University, NY

ABSTRACT:

We study a well-known class of surface models for lipid-bilayer vesicles exhibiting phase separation, incorporating a phase field together with membrane fluidity and bending elasticity. We prove the existence of a plethora of equilibria in the large, characterized by symmetry type. We overcome several difficulties in carrying this out. Obtaining the correct equilibrium equations is already a surprisingly difficult first step via the first variation of the energy. Even so, due to inherent surface fluidity combined with finite curvature elasticity, neither the Eulerian (spatial) nor the Lagrangian (material) description of the model lends itself well to analysis. This is resolved via a singularity-free radial-map description, which effectively eliminates the grossly underdetermined in-plane deformation. The resulting governing equations comprise a quasi-linear elliptic system with nonlinear constraints. We combine well known group-theoretic ideas for symmetry breaking with global bifurcation theory to obtain our existence results. We also employ a radial-map formulation within a non-local finite-element shell model to compute reliable solutions exhibiting icosahedral symmetry. We use numerical continuation to explore the behavior of solutions as the naturally occurring parameters in the model are varied, e.g., relative bending stiffness, average phase-field value, etc.

BIOGRAPHY:

Tim Healey works at the interface between the mechanics of nonlinearly elastic structures and solids and mathematical analysis. At the start of his career he pioneered the use of group-theoretic methods in global bifurcation analysis, leading to (i) efficient numerical methods in computational bifurcation and (ii) detailed global analyses of nonlinear elliptic PDEs with symmetry. He is also well known for the development of a Fredholm degree leading to the existence of solutions "in the large" in nonlinear continuum mechanics, and for developing models of chirality in Cosserat rod theory. Most recently he is focused on the modeling and analysis of thin elastic surfaces — in particular, wrinkling in highly stretched thin sheets, and pattern formation in fluid-elastic (lipid-bilayer) vesicles.

Tim Healey holds engineering degrees from the University of Missouri, Columbia (BS 1976) and the University of Illinois, Urbana-Champaign (MS 1978, PhD. 1984), during which time he studied mathematics, civil engineering, theoretical mechanics and mathematics – in that order. From 1978-80, before his PhD studies, he was a licensed engineer at a consulting firm in the Los Angeles area. He spent one year as a visiting professor of Mathematics at the University of Maryland before joining the Cornell faculty in 1985. At Cornell he has held positions in the Department of Theoretical & Applied Mechanics (1985-2008), including Chair of the Department (2000-2008), and a joint appointment in the Departments of Mathematics and Mechanical & Aerospace Engineering (2009-2014). Currently he holds a full-time appointment in the Department of Mathematics. Throughout his career he has given numerous invited presentations and key-note addresses, and has enjoyed nearly continuous support from the National Science Foundation for his research. He is the recipient of 4 teaching prizes at Cornell. He serves on several editorial boards, and has served in various leadership roles on committees and organizations representing the science of mechanics. He has held numerous visiting positions throughout his career, including distinguished visiting professorships at Universidad de Los Andes (Bogotá, Colombia) and Ecole Polytechnique (Palaiseau, France).